Real-Time Monitoring of the Freshwater Discharge at the Head of the St. Lawrence Estuary

Daniel Bourgault[1] and Vladimir G. Koutitonsky
INRS-Océanologie
Rimouski, Québec

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ABSTRACT *The freshwater discharge at the head of the St. Lawrence Estuary near Québec city in eastern Canada, was monitored at monthly timescales from 1955 to 1988 by the Quebec Department of Environment and Fauna (DEF). Since 1988, these estimates have been discontinued. Using the 1962–1988 data, two models are developed to estimate the freshwater discharge at the head of the Estuary. The first is a regression model which estimates the discharge at monthly timescales using sea level data available at Neuville near Québec city. A second order polynomial fits the data with a correlation coefficient of $R = 0.93$. The second model is a one-dimensional numerical model which estimates the hourly discharge using hourly sea level data available at Neuville and at Lauzon as upstream and downstream boundary conditions respectively. The numerical model is calibrated with current measurements for the tidal variability, and with the DEF's estimates for monthly means. The linear least-squares fits between the monthly-averaged numerical model's results and the DEF's estimates is $R = 0.91$. Although slightly lower than the regression model for monthly means, the numerical model covers a much wider range of timescales, and can be used in real-time. Analysis of long term hourly river discharge shows that the fortnightly variation of river discharge represents roughly 7% of the annual mean.*

RÉSUMÉ *Le débit d'eau douce mensuel moyen à la tête de l'estuaire du Saint-Laurent, c'est-à-dire à la hauteur de la ville de Québec dans l'est du Canada, était estimé par le Ministère de l'environnement et de la faune (MEF) du Québec de 1955 à 1988. Depuis 1988, ces estimés ne sont plus disponibles. En utilisant les données de 1962 à 1988, deux modèles ont été développés afin d'estimer l'apport d'eau douce à la tête de l'estuaire. Le premier est un modèle régressif permettant d'estimer le débit mensuel moyen à partir de données de niveau d'eau à Neuville, près de la ville de Québec. Une équation de deuxième ordre relie les données avec un coefficient de corrélation de $R = 0{,}93$. Le second est un modèle numérique unidimensionnel permettant d'estimer le débit horaire en utilisant les données de niveau d'eau horaire à Neuville et Lauzon comme conditions frontières amont et aval respectivement. Le modèle est calibré avec des observations de courant pour la variabilité semi-diurne, et avec les estimations du MEF pour les moyennes mensuelles. Le coefficient de corrélation linéaire, au sens*

[1]Now at: Department of Atmospheric and Oceanic Sciences, McGill University, Montreal, Canada

des moindres carrés, entre les estimés du MEF et ceux effectués avec l'aide du modèle numérique est de R = 0,91. Bien que légèrement inférieur pour les moyennes mensuelles que les estimations du modèle régressif, le modèle numérique permet de couvrir une plus grande variabilité d'échelle temporelle, et peut être utilisé en temps réel. L'analyse à long terme du débit fluvial horaire montre que la variation bimensuelle représente approximativement 7 % de la moyenne annuelle.

1 Introduction

The St. Lawrence River has a drainage basin of about 1.3×10^6 km^2 and, in terms of freshwater runoff into the North Atlantic Ocean, its discharge is the second largest in North America after the Mississippi River. The amount of freshwater from the St. Lawrence River that discharges into the Estuary and the Gulf of St. Lawrence controls the intensity of the buoyancy-driven circulation and of the Gaspé current, mixing processes, sea-ice distribution, and the dynamics of the ecosystem in a wide range of timescales (e.g., Koutitonsky and Bugden, 1991). In addition to natural variability, the hydrologic cycle may be modified by the regulation of rivers or by global greenhouse warming. For example, Mortsch and Quinn (1996) linked the results of global climate models to a hydrologic model of the Great Lakes Basin. They suggested that the mean annual outflow of the St. Lawrence River at Montreal could decrease by 40% in a $2 \times CO_2$ scenario. The impact of such forcing on the regulation of oceanographic processes in the St. Lawrence system and on the marine ecosystem is a challenging problem yet to be studied (Slivitzky, 1993; Mortsch and Quinn, 1996). Continuous monitoring of the physical properties of the St. Lawrence system, including continental freshwater runoff, may thus provide some indication of such global changes over the years. This is one of the objectives of the Global Ocean Observing System (GOOS) program initiated in 1990 by the International Oceanographical Commission. GOOS seeks to provide observations of the physical, chemical and biological processes over the world's oceans. These observations are needed for oceanic and atmospheric forecasting, for ocean management, and for the needs of global change research.

Mean monthly estimates of the freshwater discharge at the head of the St. Lawrence Estuary near Québec city (Fig. 1), were available from the Quebec Department of Environment and Fauna (DEF) from 1955 to 1988. The method used is described by Carrier (1976). It consists of adding the measured monthly mean discharges of the main tributaries upstream from Québec city to those of smaller tributaries. The latter ones were estimated from watershed area to discharge ratios inferred from the large tributaries. This method needed considerable data processing and, as of 1988, the DEF ceased to compute these estimates (D. Lapointe, personal communication).

The objective of this work is to provide a new means for estimating: (1) the mean monthly freshwater discharge at Québec city, based on a stage-discharge relationship, and (2) the real-time hourly discharges, based on a one-dimensional

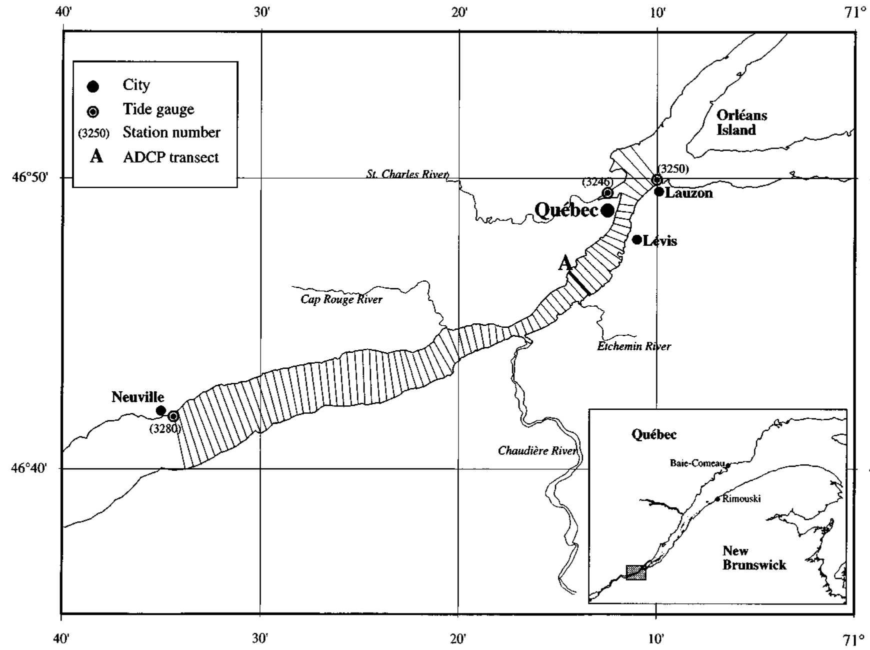

Fig. 1 Map of the head of the St. Lawrence Estuary. For the numerical model application, the river is discretized into 80 cross-sections separated by $\Delta x = 463$ m, as depicted by the thin cross-channel solid lines. The model is forced at its boundaries with water-level observations available at Neuville and Lauzon. The thick solid line (identified by the letter A) represents the cross-section where current measurements over two semi-diurnal tidal cycles were collected by Environment Canada in October 1995.

prognostic river model constrained at the upstream and downstream boundary from real-time water level observations available at Neuville and Lauzon (Fig. 1). Section 2 presents the stage-discharge relation for the mean monthly estimates of freshwater discharge. The physics underlying the numerical model developed for real-time estimates is described in Section 3. Section 4 presents the calibration and the application of the model to the head of the Estuary. Finally, Section 5 outlines the procedure for achieving real-time solutions.

2 Mean monthly freshwater discharge

a *Method*

Given a spatially uniform flow, the discharge at a cross-section of a river can be directly related to the water level at that section. The water level, or stage, being much easier to measure than the discharge, it is more practical to convert stage measurements into discharge from a stage-discharge relationship. Several theoretical stage-discharge relations derived from Manning's equation exist for canals of simple geometry or hydraulically controlled by installations such as

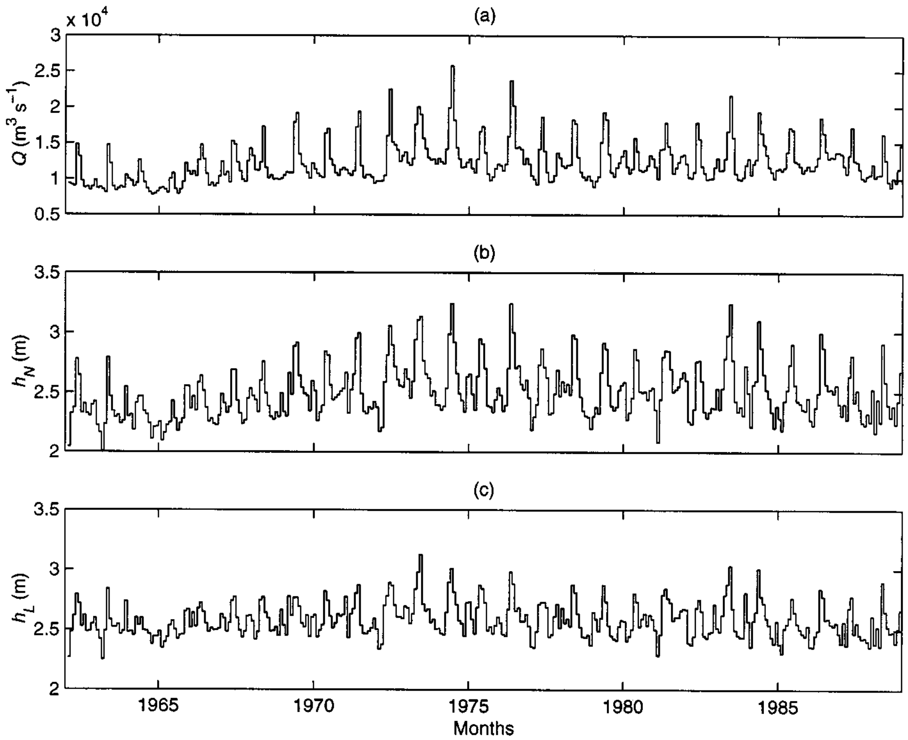

Fig. 2 DEF's monthly mean discharge at (a) the Québec-Lévis cross-section, and monthly mean water level, at (b) Neuville, and (c) Lauzon, from January 1962 to December 1988.

weirs or flumes, and can be found in most hydraulic texts (e.g., Rantz, 1982a). However, for a natural river system such as the St. Lawrence, the stage-discharge relation must be derived graphically (e.g., Graf, 1993). Once simultaneous records of stages and discharge are measured at one cross-section of a river, a regression model can be constructed and used for future estimates of the discharge. The graph showing the stage-discharge relation can take many different forms but is commonly parabolic (Grover and Harrington, 1966). The discharge of the tributaries of the St. Lawrence River are monitored from such empirical stage-discharge (see Ministère de l'Environnement et de la Faune Québec, 1997).

b *Mean Monthly Estimates*

Figure 2a shows the hydrograph of the DEF's estimates at the Quebec-Levis section (Fig. 1) from January 1962 to December 1988 computed using Carrier's method. The 30-year monthly climatological values (1959–1988) are given in Table 1 along with their standard deviations and extremes.

Table 1 depicts essentially two flow regimes over the annual cycle; high river

TABLE 1. Monthly climatological mean discharge ($m^3\ s^{-1}$) of fresh water of the St. Lawrence River at Québec city, along with the standard deviation and extremes.

Month	Mean	Min. (year)	Max. (year)
January	10 590 ± 971	8 740 (1959)	12 368 (1974)
February	10 687 ± 1 334	8 523 (1965)	14 158 (1981)
March	11 849 ± 2 357	8 103 (1965)	17 629 (1973)
April	17 025 ± 2 583	10 040 (1965)	23 723 (1976)
May	16 186 ± 3 654	10 808 (1965)	22 519 (1972)
June	12 515 ± 2 219	8 576 (1965)	18 217 (1974)
July	11 175 ± 1 857	7 938 (1965)	14 732 (1972)
August	10 727 ± 1 536	8 129 (1964)	14 132 (1976)
September	10 480 ± 1 302	7 822 (1964)	13 007 (1986)
October	11 035 ± 1 666	7 938 (1964)	13 690 (1986)
November	11 584 ± 1 638	8 243 (1964)	14 703 (1988)
December	11 359 ± 1 476	8 665 (1964)	14 042 (1979)
Annual mean	10 908 ± 1 114	9 225 (1964)	14 835 (1974)

Statistics were computed from the DEF's estimates from January 1959 to December 1988

flow is observed from April to June (≈ 15 000 $m^3\ s^{-1}$), while the rest of the year is characterized by lower flow (≈ 11 000 $m^3\ s^{-1}$). Table 1 and Fig. 2a also depict large interdecadal variabilities. For example, the amount of freshwater that discharged into the St. Lawrence Estuary in April 1976 was more than twice as much as in April 1965. In general, the mid-sixties were dry, and the mid-seventies were wet.

Long-term hourly water level observations since 1962 at the head of the Estuary are available for Neuville and Lauzon (Fig. 1) from the Marine Environmental Data Service, Ottawa. A considerable amount of data are missing in both time series. The total amount of missing data represents 16% for Neuville and 6% for Lauzon. Some of the missing segments constitute very long series. At Neuville three consecutive years, from January 1966 to January 1969, and eight consecutive months, from September 1978 through April 1979, are completely missing. At Lauzon three consecutive months, from June to September 1984, are completely missing. Other missing segments constitute a few hours to a few days or weeks. The start and end times of each missing segment of both series is given by Bourgault (1996). In order to perform statistics and to provide continuous forcing boundary conditions for the numerical model (Section 4) all missing data are interpolated. The method used is described in detail by Bourgault (1996) and is summarized in Appendix A.

Although the water level at the head of the Estuary is dominated by the mixed barotropic tide, the flow is considered steady over a monthly timescale. The monthly mean water levels at Neuville and Lauzon are computed over the same period when discharges are available, and are shown on Figs 2b and 2c, respectively. Visual

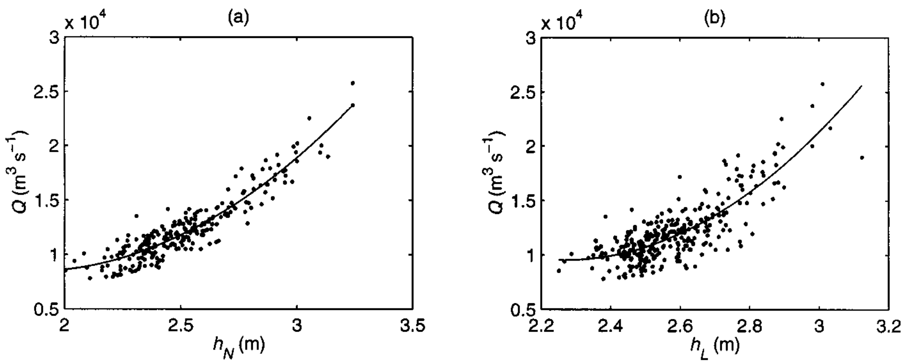

Fig. 3 DEF's monthly mean discharge plotted versus monthly mean water-level, at (a) Neuville and (b) Lauzon. The solid lines are the second-order polynomial least-squares fit as described by equations 1 and 2 respectively. Months with more than 10% missing data in the original water level series are discarded from the analysis, which results in 245 and 296 monthly averages for Neuville and Lauzon, respectively.

inspection suggests some correlation between the water level and the discharge on monthly timescales. The water level at Neuville shows greater variability than that at Lauzon. This difference is expected because in shallower regions the water level is more sensitive to variations of the discharge due to the non-linear character of the bottom friction. The cross-sectional mean water depth at Lauzon is about three times that at Neuville (see Fig. 4a) and thus, a better correlation between the discharge and the water level is expected at Neuville.

Figures 3a and 3b show the stage-discharge relationships defined by plotting the DEF's estimates of monthly mean discharge against the monthly mean water levels at Neuville and Lauzon, respectively. Any months with more than 10% of interpolated data are discarded from the analysis. A least-squares second-order polynomial is fitted through the data and the corresponding relations are given by

$$\bar{Q} = 7\,822\bar{h}_N^2 - 28\,843\bar{h}_N + 34\,999 \text{ c.i. } (95\%) = \pm 2\,099 \quad (1)$$
$$\text{for } 2.00 \leq \bar{h}_N \leq 3.24$$

and

$$\bar{Q} = 21\,976\bar{h}_L^2 - 99\,642\bar{h}_L + 122\,480 \text{ c.i. } (95\%) = \pm 3\,207 \quad (2)$$
$$\text{for } 2.25 \leq \bar{h}_L \leq 3.12$$

where the subscripts N and L refer to Neuville and Lauzon, respectively. The monthly mean discharge $\bar{Q}$ is given in $m^3\ s^{-1}$, and the monthly mean water level $\bar{h}$ is given in metres referenced to the nautical chart datum. The correlation coefficients

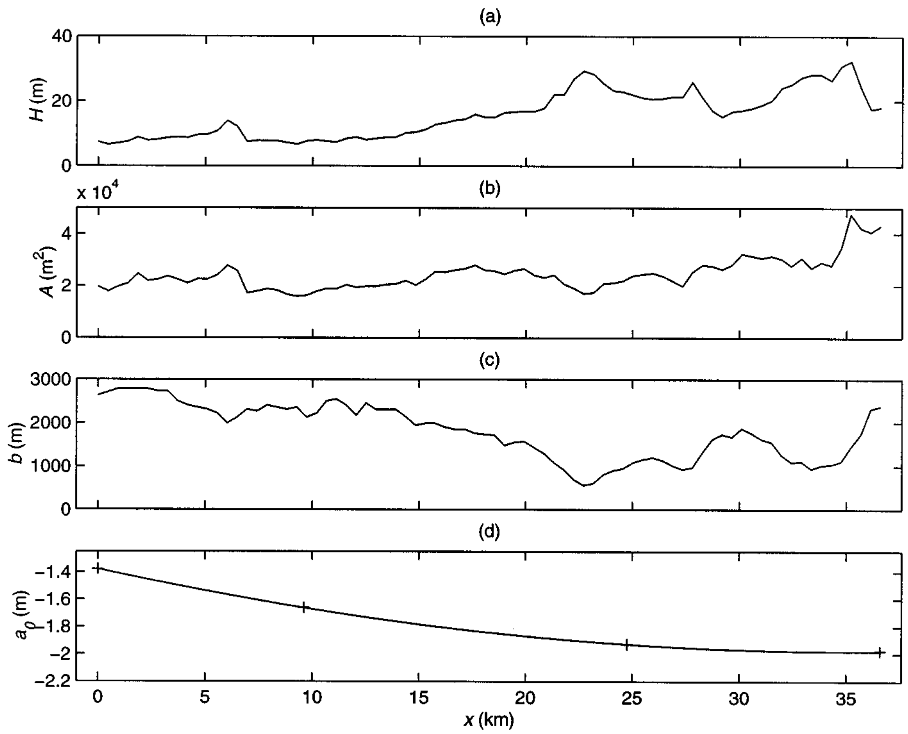

Fig. 4 Hydraulic characteristics for each of the 80 cross-sections that discretized the river between Neuville and Lauzon: (a) mean depth H; (b) area A; (c) surface width b; (d) mean water-level relative to the International Great Lakes Datum 1985 a_0. The mean water-level a_0 is interpolated at each grid cell from available measurements at Neuville, St. Augustin, Irving Wharf and Lauzon, identified by a plus sign on the figure.

are $R = 0.93$ and $R = 0.83$ for equations l and 2, respectively. The residuals (DEF's minus model estimates) are normally distributed around zero so the 95% confidence interval (c.i.) is given by $\pm 1.96\sigma$, where σ is the standard deviation of the residuals. The confidence interval gives the interval in which relations l and 2 fall within the DEF's estimates. The error of the DEF's estimates is discussed in more detail in Section 4.

By using equation 1 or 2 one only needs to compute the monthly mean water level at Neuville or Lauzon to obtain an estimate of the monthly river discharge at the head of the Estuary. Equation 1 confirms the greater sensitivity of the water level to discharge at Neuville. Other effects may also influence the lesser accuracy of equation 2, the presence of large tidal flats, the proximity of Orleans Island, and the curvature of the river are all features that most likely tend to violate the spatially uniform assumption underlying equation 2.

3 One-dimensional numerical model

The spatially uniform assumption underlying the applicability of equations 1 and 2 is no longer valid when considering the variation of the discharge at tidal (1 day) and synoptic (2–15 days) periods. Under unsteady forcing, the water level at only one cross-section of the river is not sufficient to estimate the discharge since the time variation of the hydraulic slope must also be taken into account in the calculation.

The problem of evaluating the unsteady discharge can be approached empirically or dynamically. The empirical approach includes the method of cubatures, the rating-fall method, the tide-correction method or the coaxial graphical-correlation method. These methods are useful for rough estimates but their accuracy is generally inversely proportional to the degree of unsteadiness of the flow in consideration. A detailed description of these methods is given by Rantz (1982b). If long time series of discharge at tidal frequency were available, these methods could be considered for the St. Lawrence River. However, such long term high-frequency observations do not exist. The only hourly discharge data available were obtained at cross section A (Fig. 1) by Environment Canada (J.-F. Cantin, personal communication) using an acoustic Doppler current profiler for a period of 24 hours starting at 1445 Eastern Standard Time on 11 October 1995. This time series is far too short to yield any significant empirical relation. In order to compute the continuous variation of the discharge, the hydrodynamical method is much more accurate (Rantz, 1982b) and is introduced here. The discharge is computed by solving numerically the unsteady one-dimensional St-Venant equations of momentum and continuity, using available hourly measurements of water level at Neuville and Lauzon as boundary conditions.

The currents at the head of the Estuary are dominated by the action of the mixed barotropic tide. The wavelength of this tidal wave ($\sim 6 \times 10^5$ m) is much larger than the river width ($\sim 10^3$ m) and the river depth (~ 20 m), so that the tidal streams can be considered as one-dimensional and both the transverse and vertical motions can be neglected. Under these assumptions, the one-dimensional shallow-water equations considered for the numerical model are (Dronkers, 1964)

$$\frac{\partial Q}{\partial t} + 2\,\frac{Q}{A}\left(\frac{\partial Q}{\partial x} - q\right) = -gA\left(\frac{\partial h}{\partial x} + \frac{da_0}{dx}\right) - \frac{gn^2}{H^{4/3}}\,\frac{Q|Q|}{A} \tag{3}$$

$$\frac{\partial h}{\partial t} + \frac{1}{b}\left(\frac{\partial Q}{\partial x} - q\right) = 0 \tag{4}$$

where Q is the discharge in m^3 s^{-1}, and h is the water level in metres referenced to the nautical chart datum. The other parameters are (MKS units); the surface cross-sectional width b, the cross-sectional area A, the lateral input per unit length q, the mean water level relative to the geoid a_0, Manning's coefficient of friction n (s m$^{-1/3}$), and the gravitational acceleration g. The longitudinal axis x has its origin upstream, and t is time.

Equations 3 and 4 are solved over a nonstaggered grid, following a one-dimensional adaptation of the two-dimensional numerical scheme proposed by Szymkiewicz (1993). The advantage of this scheme is that all variables are calculated at the same grid points without giving rise to numerical oscillations of a wavelength twice the numerical grid size ($2\Delta x$) inherent to a nonstaggered grid. The details of the spatial discretization and some generic experiments are presented in Appendix B. We refer to Szymkiewicz (1993) for a complete discussion of the stability and accuracy of the numerical scheme.

4 Numerical model applications

The numerical model is applied to the domain between Neuville and Lauzon. This segment of the river is discretized into 80 cross-sections separated by a grid size $\Delta x = 463$ m (Fig. 1). Figures 4a–c show the mean depth, the area and the surface width of each of the 80 cross-sections as extracted from the Canadian Hydrographic Service nautical charts 1315 and 1316. Figure 4d shows the mean water level a_0, relative to the International Great Lakes Datum 1985 (1992), interpolated at each grid section from available measurements at Neuville, St-Nicholas, Irving Wharf and Lauzon (B. Labrecque, personal communication). Hourly water elevations are prescribed at both boundaries from observations at tide gauges 3280 and 3250. Freshwater inputs from the St. Charles River, Cap Rouge River and Etchemin River are neglected. The Chaudière River is the most important tributary for this segment of the River but contributes only 1% of the annual discharge of the St. Lawrence River. In order to minimize errors, available daily climatological discharge values for the Chaudière River are used as lateral input at the sixtieth node from Neuville. The calibration consists of adjusting Manning's coefficient of bottom friction n to obtain the best fit to the observations.

a *Results*

A first simulation is carried out for the period 1445 EST 11 October to 1300 EST 12 October 1995, which corresponds to the period for which hourly discharge data are available from Environment Canada. The resulting Manning's coefficient is $n = 0.023$ s m$^{-1/3}$ and is comparable to values calculated from other authors for the same region (Morse, 1990; Prandle and Crookshank, 1972).

Figure 5a compares the model elevation with observations measured at the tidal gauge 3246 situated at Quebec harbour (Fig. 1). The tidal gauge 3246 is too close to the downstream boundary to provide a significant comparison, but it provides the only means for elevation comparison. At least, Fig. 5a shows that the model gives a smooth, consistent solution. Figure 5b shows the computed discharge at section A compared with observations. The model results show good agreement with the observations for this low flow regime period. Observations at a higher flow rate (April–June) would be needed to further assess the skill of the model. Figure 5c shows the balance of terms of the momentum equation at section A: the balance is achieved between the inertial term, the pressure gradient and bottom friction while advection is negligible.

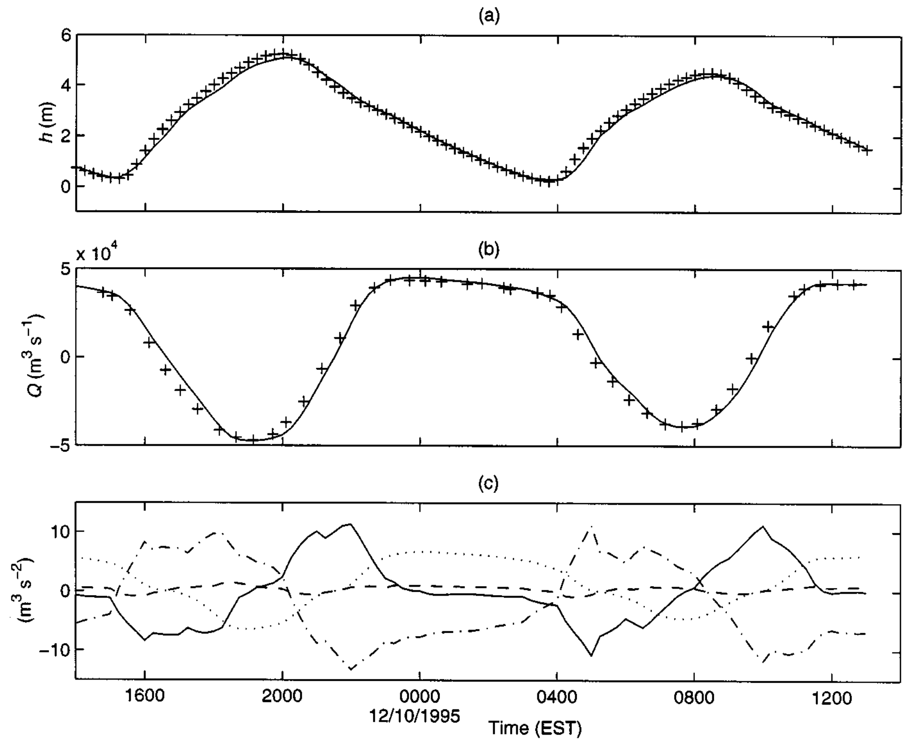

Fig. 5 (a) Comparison between observed (plus sign) and computed water level (solid line) at tidal gauge 3246 (Québec harbour). (b) Comparison between observed (plus sign) and computed discharge (solid line) at cross-section A (see Fig. 1). (c) Balance of terms of the momentum equation at cross-section A (solid line for the inertial term, dashed line for the advection term, dashed-dotted line for the pressure gradient, and dotted line for the bottom friction term).

The next simulation is carried out for the period January 1962 to December 1988, generating 27 years of hourly discharge values. These values are averaged over monthly intervals and compared with DEF's estimates. Figure 6 shows this comparison, with the differences between the two methods. Again, good overall agreement is observed, although some punctual large differences are observed (e.g., in December 1978 and November 1982). These differences are attributed to dubious water level interpolation of missing data for these months. Figure 7 shows the least-squares fit between DEF's and model estimates. The correlation coefficient is $R = 0.91$. Although the fit is slightly lower than the regression model presented in Section 2b, the numerical model covers a much wider range of timescales and can compute the discharge variability from tidal to decadal periods.

Apart from errors arising from water level interpolation, monthly differences are initially dependent on errors associated with the DEF's monthly estimates. These may originate from different sources. Firstly there are errors arising from stage-

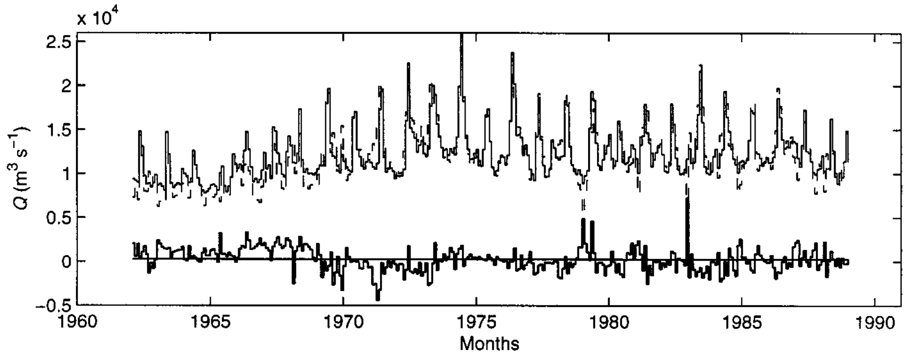

Fig. 6 Comparison between the DEF's monthly mean estimates (thin solid line), and monthly-averaged discharge from the 1D model (dashed line), along with the differences between the two methods (DEF minus 1D Model) (thick solid line), from January 1962 to December 1988.

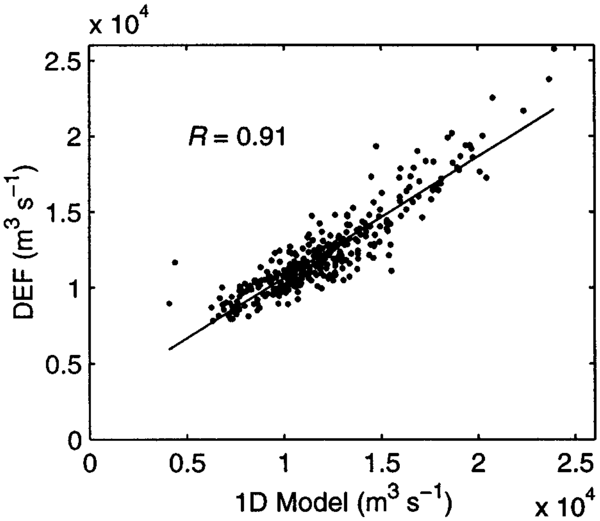

Fig. 7 DEF's mean monthly estimates versus monthly-averaged discharge from the 1D model. The solid line represents the least squares linear regression fit, along with the coefficient of correlation $R = 0.91$. The few major outliers are attributed to errors in the interpolated data over missing segments.

discharge relationships used for estimating the discharge of the largest tributaries. According to J. Laroche (personal communication), this error is about 4%. Secondly, there are errors made by estimating the discharge of smaller tributaries using the ratios obtained from the large tributaries; according to Carrier (1976) this error is around 2.5%. Thirdly, some error is introduced in the estimates by neglecting the advection time from the tributaries to the head of the Estuary. Again, according to

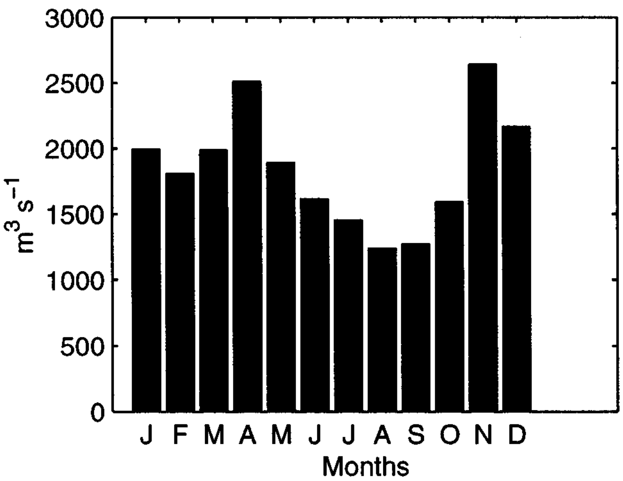

Fig. 8 Seasonal root-mean-square differences (m^3 s^{-1}) between DEF's estimates and monthly-averages from the 1D model. Statistics are computed from January 1962 to December 1988.

Carrier (1976), this error is negligible on monthly timescales. Figure 8 shows the root-mean-square monthly differences between DEF's and model estimates. The smallest differences occur during summer months (July–September) when river flow is at its minimum. The largest differences occur during months of sudden change as in April (snow melt) and November (highest precipitation). The additional surface stresses induced by sea-ice are neglected in the present model, and may explain the higher differences during winter months (December–March).

Finally, some errors in DEF's estimates may be introduced by omitting the potential effects of long-waves to raise the water level and retain (or to fall and flush) a certain volume of fresh water into the river. This has been suggested by Godin (1979) with regards to the neap-spring tidal cycle (the MS_f cycle). This can be appreciated by inspecting Fig. 9 which shows the time evolution of hourly discharge over a typical summer month (August 1988). To isolate the fortnightly component of the discharge, a harmonic analysis was performed over the year 1988 using Foreman (1977) software. The resulting MS_f amplitude is $Q_{MS_f} = 762.0$ m^3 s^{-1}, and represents roughly 7% of the annual mean. The volume of water V that is held in (or flushed out of) the River can be estimated by summing over half an MS_f period T, that is,

$$V = Q_{MS_f} \int_0^{T/2} \sin\left(\frac{2\pi}{T}t\right) dt = Q_{MS_f}\,\frac{T}{\pi} = 3.1 \times 10^8 \text{ m}^3$$

where $T = 1.275 \times 10^6$ s. It should be noted that this estimate represents an annual mean and may show considerable seasonal variations as the amplitude of

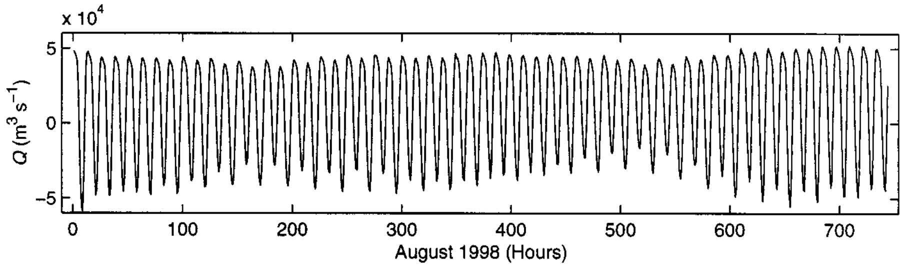

Fig. 9 Example of hourly discharge (m^3 s^{-1}) as computed by the 1D model over a typical summer month (August 1988) depicting the fortnightly modulation.

the MS_f cycle is sensitive to variations of river runoff and highest amplitudes are expected during high river flow (Griffin and Leblond, 1990). Neap-spring variations in the stratification of the Estuary were already reported and linked with marine productivity (e.g., Sinclair, 1978; Savenkoff et al., 1997) and these episodes may be related to fortnightly variations of the freshwater discharge of the St. Lawrence River. Further studies are needed to assess the impact of such hydrologic forcing on the stability and the ecosystem of the Estuary.

5 Real-time monitoring

As explained in Section 4, the 1D numerical model requires water level measurements at the upstream (Neuville) and downstream (Lauzon) boundaries in order to compute the discharge at Québec. Since 1996, hourly water level data at these two stations are available to registered users, in real-time, through a water level information system called SINECO (Système d'Information des Niveaux d'Eaux Côtières et Océaniques). This system was developed by a consortium of the Canadian Hydrographic Service (Quebec region) and a private company (Hains et al., 1994). It provides on-line access to water level data from 13 tide gauges in the St. Lawrence River between Montréal and Québec city. Water level data from Neuville and Lauzon tide gauges being available in real-time through the SINECO network, can provide boundary conditions to the numerical model and hourly discharge can be computed and made available in real-time.

6 Summary

Estimates of the freshwater discharge at the head of the St. Lawrence Estuary were available from the DEF, as monthly mean values for the period 1955 to 1988. These estimates ceased to be available in 1988. To palliate this need, two models are developed here to estimate the freshwater discharge at the head of the St. Lawrence Estuary: (1) a regression model to estimate the mean monthly discharge from monthly mean water level available at Neuville near Québec city, and (2) a 1D numerical model to estimate the hourly discharge from available water level at Neuville and Lauzon.

The regression model proposed for mean monthly estimates with the 95% confidence interval (c.i.) is,

$$\bar{Q} = 7\,822\bar{h}_N^2 - 28\,843\bar{h}_N + 34\,999 \text{ c.i. } (95\%) = \pm 2\,099$$
$$\text{for } 2.00 \le \bar{h}_N \le 3.24$$

where $\bar{Q}$ is the monthly mean discharge in m^3 s^{-1}, and $\bar{h}_N$ is the monthly mean water level at Neuville in metres referenced to the nautical chart datum. The correlation coefficient is $R = 0.93$.

The 1D numerical model developed computes hourly discharges using hourly water levels from Neuville and Lauzon tidal gauges as upstream and downstream boundary conditions. The model is calibrated with discharge measured over two semi-diurnal tidal cycles, and with the DEF's mean monthly estimates for the period 1962–1988. The least-squares fit between the 1D monthly-averages and the DEF's estimates gives a correlation coefficient of $R = 0.91$. Although the fit is slightly lower than the regression model for monthly means, the numerical model covers a much wider range of timescales and can compute the discharge variability from tidal to decadal periods. It is also shown that the amplitude of the fortnightly variability (MS_f period) of the fresh water is approximately $Q_{MS_f} \approx 762$ m^3 s^{-1} and represents roughly 7% of the annual mean. The impact of such forcing on the Estuary has yet to be studied.

Water level data from the Neuville and Lauzon tide gauges, being available in real-time through the SINECO network, can be provided in real-time to the numerical model as boundary conditions and discharge can be computed in real-time. In addition, given that 30-day water level forecasts are now available from the Canadian Hydrographic Service a similar forecast for currents could be estimated at the head of the Estuary for navigational purposes, accidental spills, ice drift or other applications.

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Appendix A: Interpolation of missing data

Summarized here is the method we adopt to interpolate missing data from the water

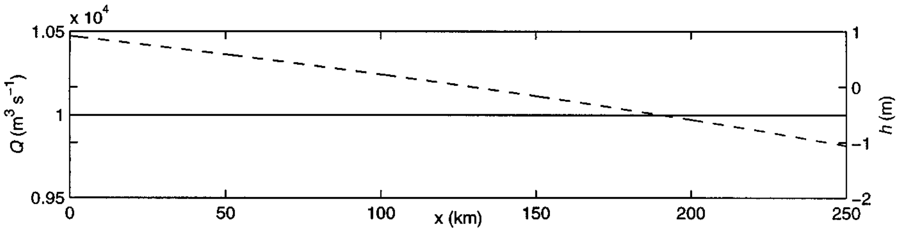

Fig. B1 Discharge (solid line) and water level (dashed line) over a hypothetical canal of simple geometry after steady state has been reached from a state at rest, and under steady boundaries forcing conditions ($Q(t) = 10^4$ m^3 s^{-1} at both boundaries).

level time series. The water fluctuations h at the head of the St. Lawrence Estuary are represented according to the following model,

$$h(t) = h_0 + h_{dm} + h_{sqp} + h_{res}$$

where h_0 is the mean water level, h_{dm} are the deterministic tidal fluctuations, h_{sqp} are the stochastic quasi-periodic fluctuations, and h_{res} are the residual fluctuations. The long-term mean water level variations h_0 are related to the seasonal variability of the river discharge. The quasi-periodic fluctuations h_{sqp} are related to meteorological perturbations characterized by periods between 2 and 15 days (Koutitonsky and Bugden, 1991). The residual fluctuations h_{res} are characterized with periods less than tidal constituents and are neglected in the interpolation.

For each missing segment of each series (Neuville and Lauzon), the following steps are applied:

1. A harmonic analysis is performed over the month preceding each missing segment using Foreman (1977) software. The result of the analysis is then used to predict the deterministic tidal fluctuations h_{dm} over the missing segments.
2. A Fast Fourier Transform filter (Walters and Heston, 1982) is applied to obtain the low-frequencies fluctuations (with period $>$ 30 hours). Low-pass fluctuations of adjacent stations are also obtained by applying the same filter. The low-pass series are then cross-correlated and least squares linear regression functions are obtained with proper time lag. The linear functions are then used to interpolate h_0 and h_{sqp} over the missing segments. When missing data are coincident, in both series, the next closest tidal gauge is used.
3. The resulting series from steps 1) and 2) are added to provide $h(t)$. The residual fluctuations h_{res} are neglected.

Following the above mentioned method we have reconstituted a 31-year (1962–1992) continuous hourly time series for Neuville and Lauzon. Although we have not quantified the exact error arising from this method, we see that, qualitatively (apart from a few exceptions, see Section 4), it provides a valuable means to obtain

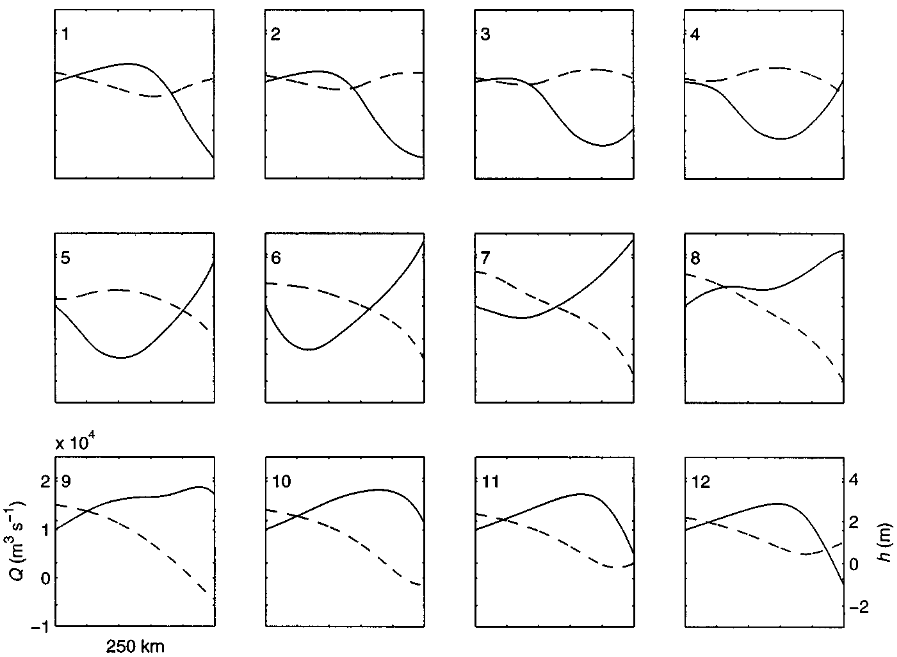

Fig. B2 Discharge (solid line), and water level (dashed line) over a hypothetical canal of simple geometry, for each hour of a semi-diurnal tidal cycle. The model is forced at its upstream boundary with a constant discharge of $Q(t) = 10^4$ m^3 s^{-1}, and at its downstream boundary with a semi-diurnal cyclical variation of the water level.

continuous water level time series, needed to perform long-term model simulations.

Appendix B: Details of the numerical scheme and generic experiments

Discretization of equations 3 and 4 is carried out in the nonstaggered grid with spatial grid size Δx. The spatial discretization is made by the approximation of the derivatives of any function ψ in the x direction at the centre increment (x_i, x_{i+1}) by,

$$\left.\frac{\partial \psi}{\partial x}\right|_0 \approx \frac{\psi_{i+1} - \psi_i}{\Delta x}$$

The variables at this point are determined as an arithmetic mean of the values at the adjacent grid point, that is,

$$\psi|_0 \approx \frac{\psi_{i+1} + \psi_i}{2}$$

The approximation in every grid cell provides the following system of ordinary differential equations for Q and h,

$$\frac{dQ_{i+1}}{dt} + \frac{dQ_i}{dt} + 2\left(\frac{Q_{i+1} + Q_i}{A_{i+1} + A_i}\right)\left(\frac{Q_{i+1} - Q_i}{\Delta x} - \frac{q_{i+1} + q_i}{2}\right)$$

$$= -g(A_{i+1} + A_i)\left(\frac{h_{i+1} - h_i}{\Delta x} + \frac{a_{0,i+1} + a_{0,i}}{\Delta x}\right)$$

$$- \frac{gn^2}{(H_{i+1}^{4/3} + H_i^{4/3})}\,\frac{(Q_{i+1} + Q_i)|Q_{i+1} + Q_i|}{(A_{i+1} + A_i)}$$

$$\frac{dh_{i+1}}{dt} + \frac{dh_i}{dt} + \frac{4}{(b_{i+1} + b_i)}\left(\frac{Q_{i+1} - Q_i}{\Delta x} - \frac{q_{i+1} + q_i}{2}\right) = 0$$

for $i = 1, 2, 3, \ldots, N - 1$. More details on the semi-implicit temporal scheme and the stability and accuracy of the method can be found in Szymkziewicz (1993).

The numerical model was first tested by means of a hypothetical canal of simple geometry. The canal has a length of 2×10^5 m and a constant width of 10^3 m. The depth is uniform at $H_0 = 15$ m. The grid size used is $\Delta x = 500$ m, so there are 500 nodes. In this first experiment, a constant discharge $Q(t) = 10^4$ m^3 s^{-1} at the upstream and downstream boundaries is prescribed. The water levels at these boundaries are free to oscillate. The time step prescribed is $\Delta t = 40$ s (Courant number: $C_r = \sqrt{gH_0}/(\Delta x/\Delta t) = 1$) and Manning's coefficient is set to $n = 0.025$ s m$^{-1/3}$. The initial condition corresponds to the hydrostatic state. Figure B1 illustrates the discharge Q and the elevation h over the domain after steady state is reached after about 4.5×10^4 s ($\approx$ 12 hours) which corresponds to 1 125 time steps. The solution for Q and h is spatially smooth without "$2\Delta x$" oscillations and the discharge is conserved over the domain.

The next experiment deals with the same canal but with different boundary conditions. At the upstream boundary, a constant discharge $Q(t) = 10^4$ m^3 s^{-1} is prescribed as in the first experiment, while at the downstream boundary, the water level is prescribed as a sinusoidal function of time: $h = a\ \sin(\omega t)$, with $\omega = 2\pi/43\,200$ s^{-1} and $a = 2.0$ m. This function approximates a tidal signal in a river with a period of 12 hours. Figure B2 illustrates the discharge and the surface elevation for each hour through one 12-hour cycle. The solution is spatially smooth without "$2\Delta x$" oscillations. The wave is damped due to bottom friction as it propagates upstream and the amplitude at the upstream boundary is about 0.25 m.

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